THE EARTH AS A SPHERE

Features and Location of Places

The Equator, Great Circle, Small Circles, Meridian, Latitudes and Longitudes

Describe the equator, great circle, small circles, meridian, latitudes and longitudes

Definition of latitude and longitude

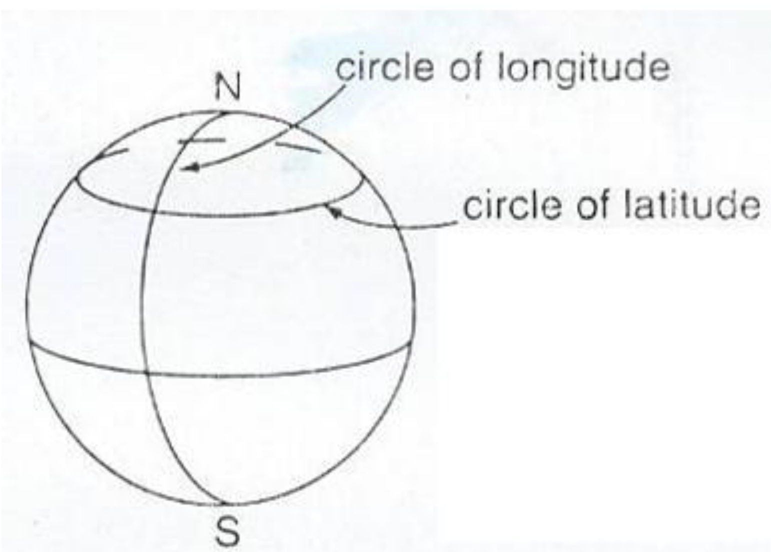
The Earth is not a perfect sphere, as it is slightly flatter at the north and south poles than at the equator. But for most purposes we assume that it is a sphere.

The position of any point on earth is located by circles round the earth, as follows:

The earth rotates about its axis, which stretches from the north to the South Pole.

Circles round the Earth perpendicular to the axis are circles of **Latitude** and Circles round the Earth which go through the poles are circles of ***Longitude*** or ***meridians*.**

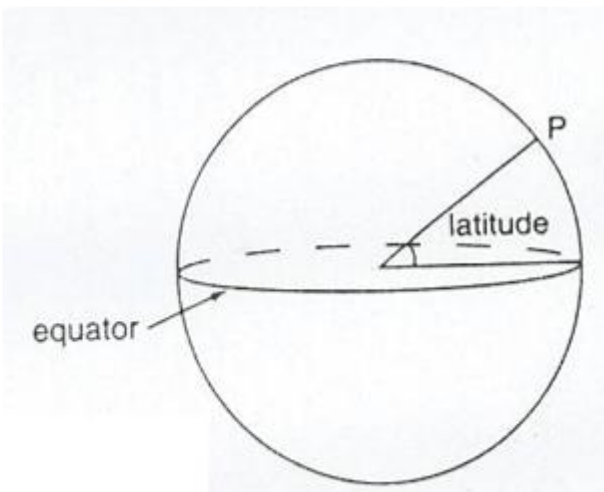
Consider the following diagram



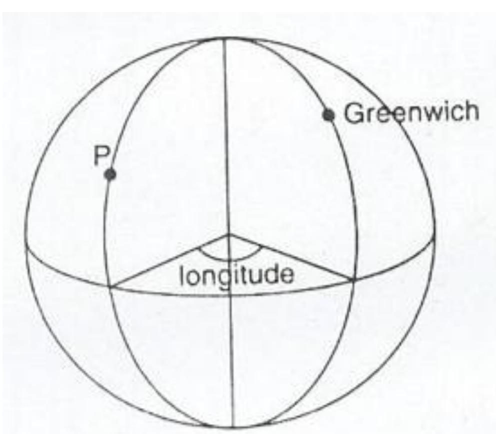
Normally ***Latitude*** is defined relative to the equator, which is the circle of latitude round the middle of the Earth while ***Longitude*** is defined relative to the circle of longitude which passes through Greenwich in London (**Greenwich meridian).**

The latitude of a position tells us how far north or south of the equator it is while the longitude of a position shows us how far east or west of the Greenwich meridian it is.

Latitude; If we draw a line from the centre of the Earth to any position P , then the angle between this line and the plane of the equator is the latitude of P.



Longitude: This is the angle between the plane through the circle of any Longitude P and the plane of the Greenwich meridian .



Latitude can be either North or South of the equator while **Longitude** can be either East or West of Greenwich.

When locating the latitude and longitude of a place we write the latitude first then longitude.

Example 1

Dar es Salaam has latitude 7°S (i.e. 7° south of the equator) and longitude 39°E (i.e. 39° east of the Greenwich meridian). So Dar es Salaam is at (7°S, 39°E).

NB; Greenwich itself has latitude 51°N (i.e. 51° north of the equator)and longitude 0° (by definition). Johannesburg has latitude 26°S (i.e. 26 south of the equator) and longitude 28°E(i.e. 28°east of the Greenwich meridian), therefore Johannesburg is at (26°S, 28°E).The north pole has latitude 90°S but its longitude is not defined. (Every circle of longitude goes through the north pole).The south pole has latitude 90°s. Its longitude is not defined. ***So all points on the equator (such as Nanyuki in Kenya) have latitude 0°***

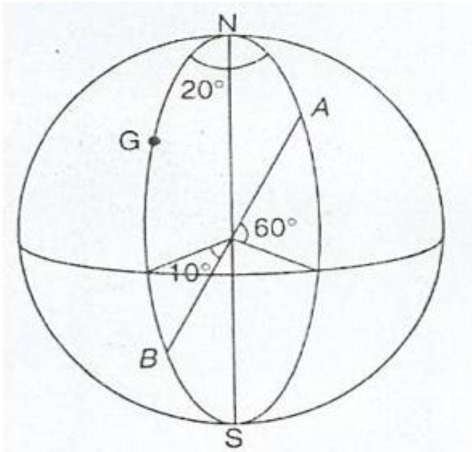
Ranges; Latitude varies between 90°S (at the south pole) to 90°N (at the north pole).

Ranges; Latitude varies between 90°S (at the south pole) to 90°N (at the north pole). **Longitude** varies between 180°E and 180°W. These are the longitudes on the opposite side of the Earth from Greenwich.

GREAT AND SMALL CIRCLES: There is an essential difference between latitude and longitude. Circles of longitude all have equal circumference. Circles of latitude get smaller as they approach the poles. The centre of a circle of longitude is at the centre of the earth. They are called ***great circles***. For circles of latitude, only the equator itself is a great circle. Circles of latitude are called ***small circles.***

Example 2

Find the latitudes and longitudes of A and B on the diagram below;



Solution;

The point A is 60° above equator, and 20° east of Greenwich.

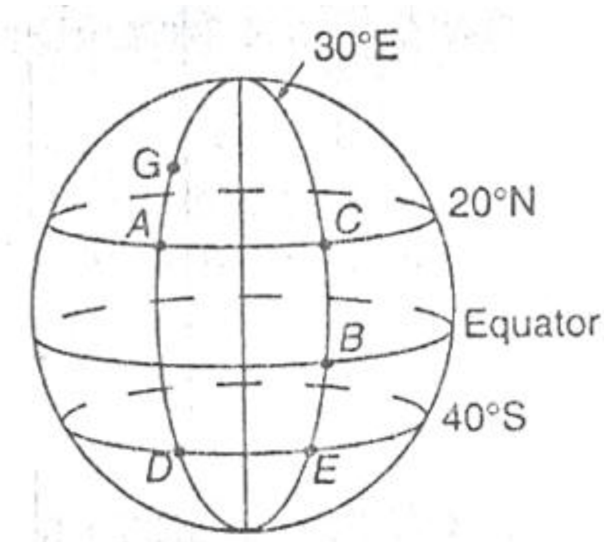
So the point A is at (60°N, 20°E)

The point B is 10° below the equator, and on the Greenwich meridian.

So the point B is at (10°S, 0°).

Exercise 1

1. Write down the latitude and longitude of the places shown on figures below:



2. Copy the diagram show on the figure above and mark these points:

a. (10°N, 30°E)

b. (20°N, 20°W)

c. (0°, 20°W)

3. Obtain a globe, and on it identify the following places.

a. (40°S, 30°E)

b. (50°S,20°W)

c. (10°N,40°W)

d. (40°N, 30°E)

e. (80°N,10°E)

f. (0°,0°)

Difference between angles of latitude or longitude

Suppose two places have the same longitude but different latitudes. Then they are north and south of each other.

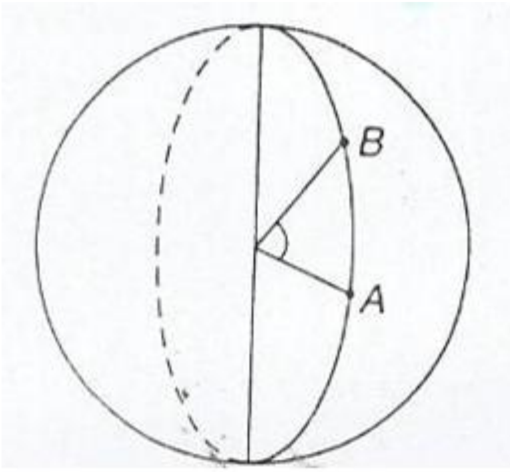
In finding the difference between the latitudes take account of whether they are on the same side of the equator or not.

- If both points are south of the equator subtract the latitudes
- If both points are the north of the equator subtract the latitudes
- If one point is south of the equator and the other north then *add* the latitudes

Similarly, suppose two places have the same latitudes but different longitudes:

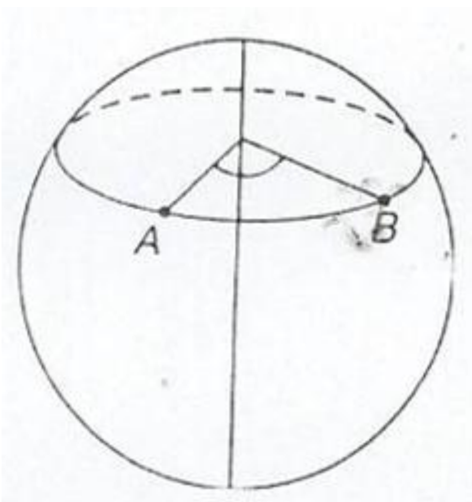
- If both points are east of Greenwich subtract the Longitudes
- If both points are west of Greenwich subtract the Longitudes
- If one point is east of Greenwich and the other west then *add* the longitudes

Suppose places *A* and *B* are on the same longitude, then the difference in latitude is the**angle** subtended by *AB* at the centre of the earth.



Suppose places A and *B* are on the same latitude.

Then the difference in longitude is the **angle** subtended by *AB* on the earth's axis.



Locating a Place on the Earth's Surface

Locate a place on the Earth's surface

Example 3

Three places on longitude 30°E are Alexandria (in Egypt) at (31°N, 30°E), Kigali (in Rwanda) at (2°S, 30°) and Pietermaritzburg (in South Africa) at (30°S, 30°E).
Find the difference in latitude between

a. Kigali and Pietermaritzburg

b. Kigali and Alexandria

Solution

(a)Both Towns are south of the equator. So subtract the latitudes. 30 – 2 = 28

Therefore the difference is 28^0

(b) Kigali is south of the equator, and Alexandria is north, so add the latitudes

31 + 2 = 33

The difference is 33^0

Example 4

A plane starts at Chileka airport (in Malawi) which is at (16°S, 35°E). It flies west for 50°. What is its new latitude and longitude?

Solution

Since it flies west, then subtract 35° from 50°. This gives 15°

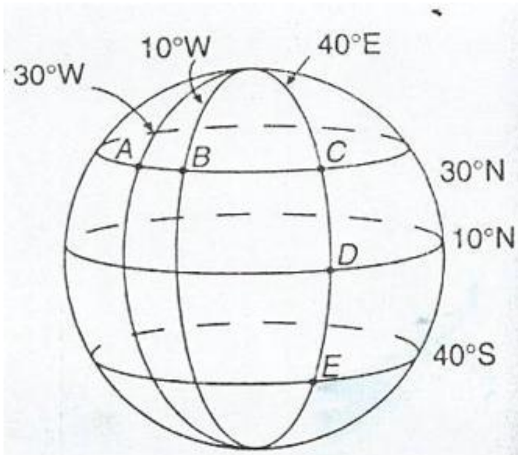
The new longitude is now west of Greenwich, hence the plane is at (16°S, 15°W).

Exercise 2

1. In the diagram shown in the following figure find,

a. The difference in longitude between A and B

b. The difference in longitude between D and E



2. Find the difference in latitude between the following pairs of places
   (a) Iringa (8°S, 36°E) and Gendi (4°S, 36°E)
   (b) Zanzibar ((6°S, 39°E) and Chiungutwa (11°S,39°E)
3. Find the difference in longitude between the following pairs of places
   a. Ibadan (Nigeria), (7°N,4°E) and Makurdi (Nigeria), (8°S,36°E)
   b. Ibadan and Kumasi (Ghana) (7°N, 2°W)
4. The following is a list of places. Find pairs of places that have the same latitude or the same longitude. For each pair with the same longitude, find the difference in latitude.

A (30°S, 20°E)	B (10°S, 50°E)	C (20°S, 40°W)
D (20°N, 40°W)	E (10°N, 50°W)	F (100°S, 50°E)
G (30°S, 40°E)	H (20°N, 10°E)	I (20°N, 40°E)
J (10°N, 30°E)	K (30°N, 40°W)	L (10°S, 10°E)

5. A plane starts at (20°N, 27°E) and flies south for 42°. What is its new latitude and longitude?
6. A plane starts at (51°N, 31°E) and flies west for 45°. What is its new latitude and longitude?

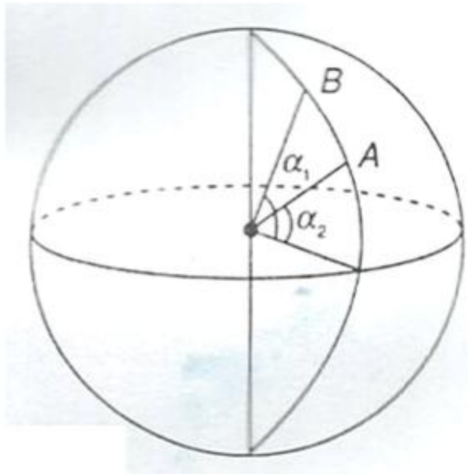
Distances along Great Circles

Distances along Great Circles

Calculate distances along great circles

Take two places X and Y on the same line of longitude, i.e. one place is due north of the other. Suppose X is due north of Y. When travelling north from Y to X, you travel along part of a circle of longitude that is you travel along an arc of the circle.

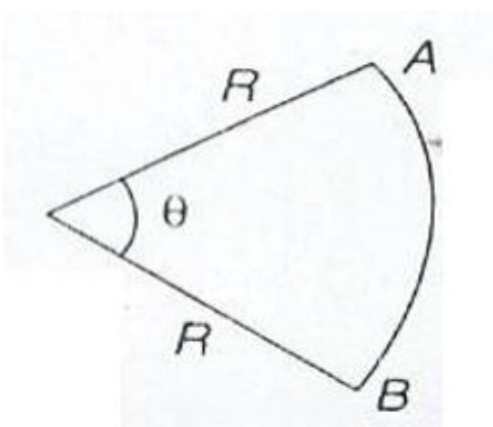
The diagram below shows two points A and B on the same circle of longitude.



The difference between their latitudes is $\theta = \alpha_1 - \alpha_2$

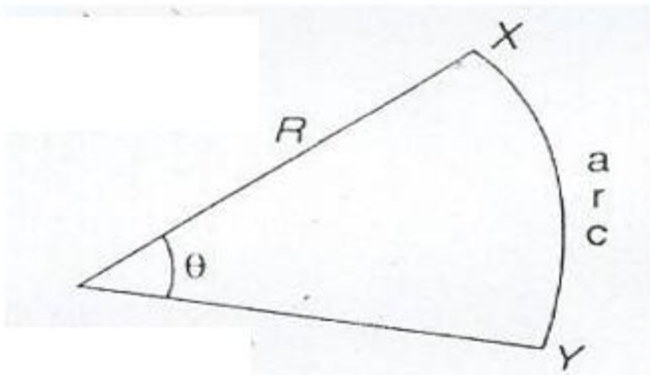
In the figure above, the sector containing the arc AB subtending∅, is shown.

Recall the formula for the length of arc.



If an arc subtends θ at the Centre of the circle of radius R, then

Arc length = $\frac{2\pi R\theta}{360}$



In the figure above R is the radius of the Earth and θ is the difference between the latitudes of X and Y. Taking R to be 6,400 km, the formula becomes

$$\text{Distance} = \frac{2\pi \times 6{,}400 \times \theta}{360} = 111.7\theta$$

NB: Remember, to find the difference in latitudes, take account of whether the places are north or south of the equator. If they are all found in south or north, then subtract the latitudes. If one is south and the other North then add the latitudes.

Nautical miles

Distances are also measured in ***nautical miles***. One **nautical** mile (nm) corresponds to one minute of latitude. Let the difference in latitudes between two places be θ.

The number of minutes in θ is 60θ, since 1 degree has 60 minutes.

Hence the distance between the places, along the arc of longitude, is 60θ nm.

So 60θ nm $= \frac{2\pi R\theta}{360}$ Kilometers

If we divide by 60θ, and take R = 6,400,

then 1nm = $\frac{2\pi \times 6{,}400}{360 \times 60}$ = 1.862 km.

1knot is speed of 1 nautical mile per hour.

Navigation Related Problems

Solve navigation related problems

Example 5

Find the distance between Alexandria (31°N, 30°E) and Kigali (2°S, 30°E)

Solution

Note that both places are on the same longitude. The difference in latitude is 33°.
Use the formula

Distance = $\frac{2\pi \times 6,400 \times 33}{360} = 3,690$

*Therefore the distance is **3,690 km.***

The difference in latitude is 33°. Hence the difference in minutes is $33 \times 60' = 1,980'$.

This is the distance in nautical miles. The distance is 1,980 nm.

Note: $1,980 \times 1.862 = 3,690,$ to 3 significant figures. Hence the two answers are the same.

Example 6

A plane starts at (20^0S, 30^0E), and flies north for 4000 km. Find its new latitude and longitude.

Solution:

The plane flies north, hence its longitude is unchanged. The plane starts south of the equator, and flying north. It may cross the equator, and so end up north of the equator. In this case the latitude south of the equator will be negative.

Suppose the plane has flown along x^0 of latitude. Then using the formula for arc length

$$4,000 = \frac{2\pi R x}{360}$$

$$x = \frac{4,000 \times 360}{2\pi R} = 35.8$$

So subtracting 35.8° from 20°S implies 20°-35.8° = -15.8°

A negative latitude south is equivalent to a latitude north. Hence the new latitude is 15.8° N therefore rounding the answer to the nearest degree the new position is (16°N, 30°E).

Example 7

A plane flies north from (10°S, 30°E) to ((27°N, 30°E) taking a time of 3 hours.

Find its speed, giving your answer I both knots and kilometers per hour.

Solution

The plane is flying along a line of longitude.

Its change in latitude is 10°+27° = 37°

The number of minutes is 60 x 37 = 2,220', so it has flown 2,220 nautical miles.

To find the speed in knots, divide 2,220 nautical miles by 3, ***the speed is 740 knots***.

Recall that 1 nautical mile is 1.862 km. So 1 knot (1 nm per hour) is equal to 1.862 km/hr.

Multiplying 740 by 1.862 gives the speed, ***so the speed is 1,378 km/hr***

Exercise 3

Consider the following Questions.

1. Find the distance in kms between the following places.
   (a) Bangalore (13°N, 78°E) and Agra (27°N, 78°E)
   (b) Asmara (15°N, 39°E) and Mombasa (4°S, 39°E)
2. A plane starts as ((30°N, 40°W) and flies 300 km north. What is its new position?
3. A ship starts at (30°N, 150°W) and sails south for 4,700 km. what is its new position?
4. A plane starts at (35°N, 90°W) and flies south for 5,590 km. What is its new position?
5. A plane flies at 600km/h. How long does it take to go from (32°N,51°E) to (5°S,51°E)
6. A ship sails at 20km/h. How long does it take to go from (31°N, 150°W) to (22°S,150°W)?
7. Find the distances in nautical miles between the places in question 1.
8. A plane starts at (10°N, 18°E) and flies north for 1,200 nm. Find its new position.
9. A ship sails south at 10.4 knots. if it starts at (15°N, 150°W), find its position after 40 hrs.
10. A Plane flies south from (12°N, 36°E) to (7°S, 36°E), taking 2½ hours. Find its speed, giving your answer in both knots and kms per hr.
11. A ship sailed north from (45°S, 28°W) to (39°S, 28°W), taking 30hrs. What was its speed? Give your answer in both knots and kms per hr.

Distances along Small Circles

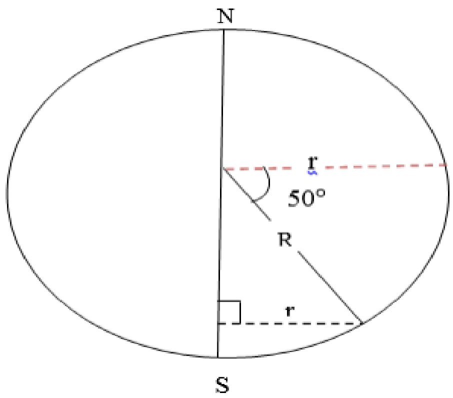
Distance along Small Circles

Calculate distance along small circles

Suppose P and Q are places west or east of each other, i.e they lie on the same circle of latitude. Then when you travel due east or west from P to Q you travel along an arc of the circle of latitude.

The situation here is slightly different from that of the previous section. While circles of longitude all have the same length, circles of latitude get smaller as they get nearer the poles.

Consider the circle of latitude 50°S. Let its radius be r km.



Then taking a point P on the circle, and letting R be radius of the earth:

$$Cos 50° = \frac{r}{R} \quad \text{or} \quad r = R \times \cos 50°$$

In general, the circle of latitude α has radius R$\cos\alpha$ .This is true for latitudes both north and south of the equator . Now the distance along a circle of latitude can be found .Suppose the difference between longitudes is θ .Then the length of arc going from P to Q is

$$\text{Arc length} = \frac{2\pi r\theta}{360} = \frac{2\pi(R\cos\alpha)\theta}{360}$$

But in kilometers $\frac{2\pi R}{360}$ = 111.7. Hence the arc length is $\boldsymbol{1117.7\,(\cos\alpha)\theta\ km}$

Nautical miles

Generally if two points on the same latitude α have difference in longitudeθ, then in nautical miles the distance along the circle of latitude is $60\boldsymbol{\theta}\mathbf{\cos}\ \boldsymbol{\alpha\ nm}$

Example 8

Find the distance in km and nm along a circle of latitude between (20°N, 30°E) and (20°N, 40°W).

Solution:

Both places are on latitude 20°N. The difference in longitude is 70°. Use the formula for distance.

Distance = 111.7 cos20° x 70°. Hence the distance in nautical miles is 60 x 70 x cos20°

The distance is ***3,950 nm.***

Example 9

A ship starts at (40°S, 30°W) and sails due west for 1,000 km. Find its new latitude and longitude.

Solution:

Because it sails due west, the latitude remains unchanged. Suppose it has sailed through x° of longitude. Use formula.

$1{,}000 = 111.7 \times \cos 40° \times x°$

Hence $x = \frac{1{,}000}{111.7 \cos 40°} = 12$

Add 12° to the longitude, obtaining 42°.

Therefore the new position is *(40°S, 42°W)*.

Example 10

A ship sails west from (20°S, 15°E) to (20°S, 23°E), taking 37 hours. Find speed, in knots and in kms per hr.

Solution:

The difference in longitude between the two points is 8°. Hence the distance, in nautical miles, is.

$60 \times 8 \times \cos 20° = 451$ nm

Divide by 37 to obtain the speed

The speed is ***12.2 knots.***

To obtain the speed in kms per hr, multiply by 1.862.

The speed is ***22.7 km/hr.***

Exercise 4

Consider the following Questions.

1. Find the distance in km between these places.
   a. Johannesburg (26°S, 28°W) and Maputo (26°S,33°W)
   b. Washington (38°N,77°W) and San Francisco (38°N,122°W)
2. A plane flies due east from (30°S,13°E) for 1,000km. What is its new position?
3. A plane flies due east from (45°N, 5°W) for 1,200 km. what is its new position?
4. A plane flies due east from (23°S,12°W) for 1,650km. What is its new position?
5. A ship sails due west from (30°S,12°E) for 2,800km, what is its new position?
6. Fins the distances in nautical miles between the places of question 1.
7. A plane starts at (40°S,30°E) and flies east for 600nm. Find its new position.
8. A plane starts at (20°N,10°W) and flies east for 720nm. Find its new position.
9. A ship starts at (40°S, 140°W), It sails north for 12 hrs, then east for 20hrs. If its speed was constant 20knots, find the latitude and longitude of its final position.
10. A ship starts at (30°N, 45°W), and sails north to (34°N,45°W). It then sails east to (34°N,41°W). if the journey took 30 hrs, find the speed, given that it was constant.
11. A ship can sail at 24km/hr. it starts at (30°S, 60°E). It sails due south for 8 hrs, then due east 2 hr. find the latitude and longitude of its final position.

Navigation

Suppose a ship is sailing in a sea current, or that a plane is flying in a wind. Then the course set the ship or plane is not the direction that it will move in. the actual direction and speed can be found either by scale or by the use of Pythagoras's theorem and trigonometry.

Draw the line representing the motion of the ship relative to the water. At the end of this line draw a line representing the current. Draw the third side of the triangle. This side, shown with a double – headed arrow, is the actual course of ship.

Example 11

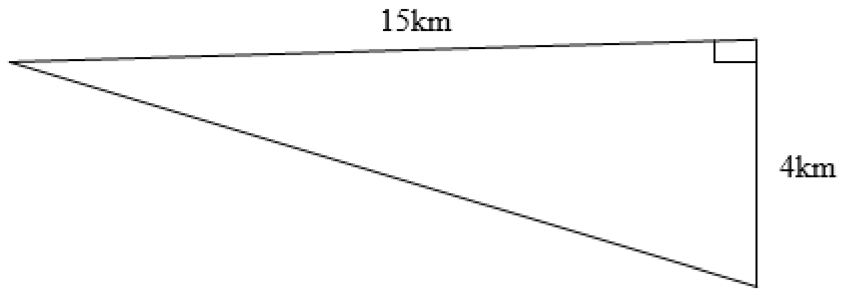
A ship sets course due east. In still water the ship can sail at 15km/hr. There is a current following due south of 4kkm/hr. use a scale drawing to find.

a. The speed of the ship

b. The bearing of the sip.

Solution:

In one hour the ship sails 15km east relative to the water. Draw a horizontal line of length 15cm. In one hour the current pulls the ship 4km south. At the end of the horizontal line, draw a vertical line of length 4cm.



(a) Measure the third side of triangle as 15.5cm.
The speed of the ship is ***15.5 km/hr.***

(b) Measure the angle between the course of the ship and east as 15°. Add this to 90°
The ship is sailing along a bearing of 105°

Note: These results can also be found by Pythagoras' theorem and trigonometry.
Thus the speed is $\sqrt{15^2 + 4^2}$, and the angle with east is tan $tan^{-1}\left(\frac{4}{15}\right)$

Example 12

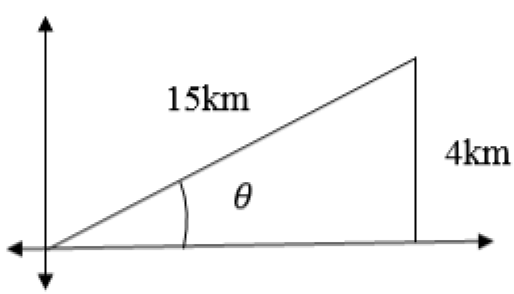
The ship of example 10 needs to travel due east. Calculate the following.

a. What course should be set?

b. How long will the ship take to cover 120km?

Solution

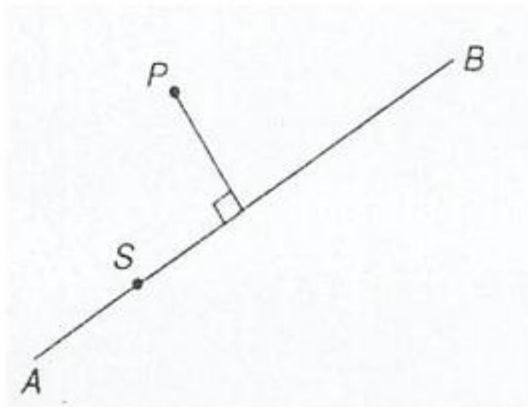
The ship needs to set a course slightly north of east, consider the following diagram.



(a) By trigonometry:
Sin $\theta = \frac{4}{15}$, hence $\theta = 15.46°$, Substract this from 90°
A course of ***074.5 º*** should be set.

(b) Using Pythagoras' theorem in triangle,
Actual speed of the ship = $\sqrt{15^2 - 4^2}$ = 14.46 km/hr
Now divide 120by this speed.
The ship will take ***8.3 hrs***.

Note: With no current, the journey would take 8hrs. The journey takes slightly longer when there is a current.Suppose a ship or a plane does not directly reach a position. We can still find how close the ship or plane is to the position.



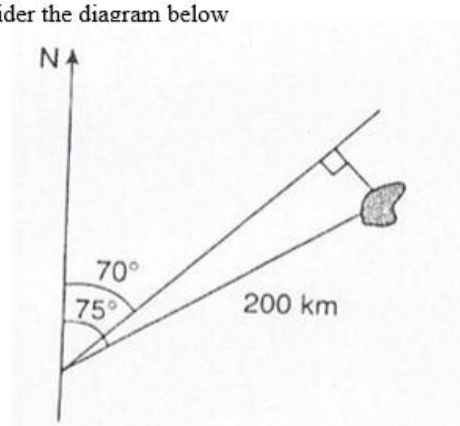
In the diagram above, the ship S (or plane) is travelling on a straight line AB.
The shortest distance to point P is when SP is perpendicular to AB.

Example 13

A small island is 200km away on a bearing of 075°. A ship sails on a bearing of 070°.Find the closest that the ship is to the island.

Solution:

Consider the diagram below



The angle between the directions of the island and the path of the ship is:

$75° - 70° = 5°$

When a ship is closest to the island, the line from the island to the ship is at 90° to the path of the ship, hence, by trigonometry,

$d = 200 \times \sin 5° = 17.4$km.

The closet distance is ***17.4km.***

Exercise 5

1. Find the difference in longitude between Cape Town (34°S, 18°E) and Buenos Aires (34°S, 58°W)

2. A ship startsat (15°N, 30°W) and sails south for 2,500km. Where does it end up?

3. Find thedistance in km along circle of latitude between cape Town and Buenos aires (see question 2)

4. A plane starts at (37°S, 23°W) and flies east for 1,500 km. where does it end up?

5. Find thedistance in nautical miles between the places in question 1.

6. A plane startsat (43°N, 17°E) and flies west for 960nm. Where does ie end up?

7. A sea current of6 km/hr flows due east. A ship that can sail at 19 km/hr in still water sets a course due south. Find the following by scale drawing.

a. The direction in which the ship sails

b. The actual speed of the ship

8. The ship of question 4 needs to travel due south. How long will the ship take to travel 260 km

9.A plane flies on a bearing of 342°.There is a control tower 100 km from the plane on a bearing of 350°. Find the shortest distance between the plane and the control tower.

10. A lighthouse is 150 km from ship on a bearing of 123°. The ship sails on a bearing of 138°. Calculate the shortest distance between the ship and the lighthouse